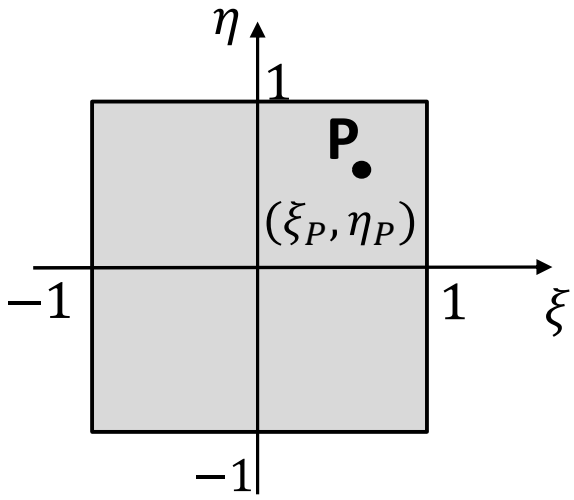
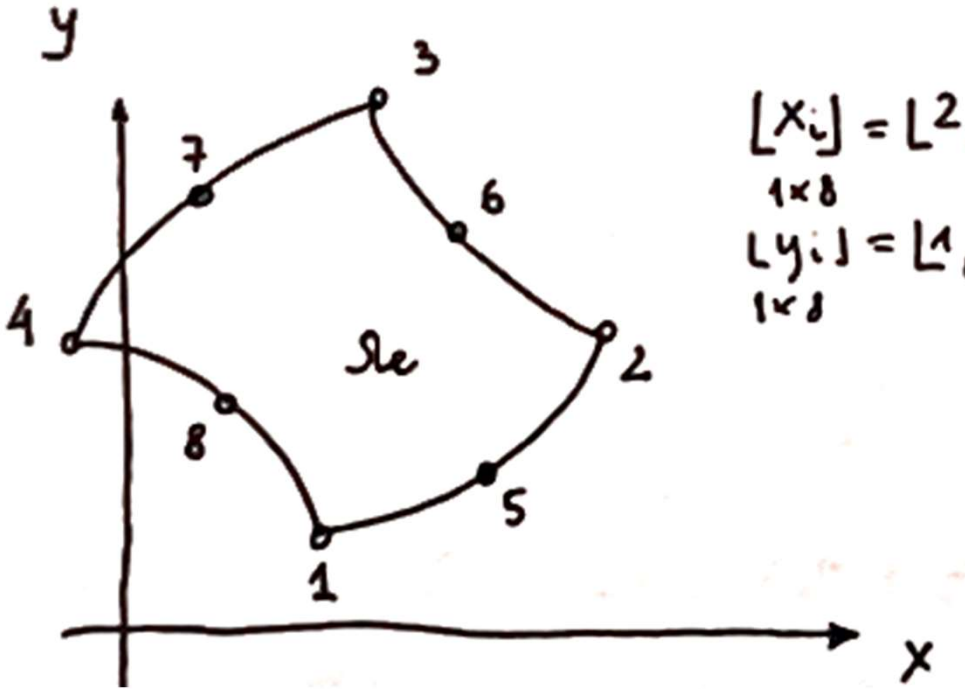
Example. QUAD-8node: Find coordinates and det [J] at point P.

$$\xi_P = \eta_P = \frac{1}{\sqrt{3}}$$





$[x_i] = [2, 7, 4, -1, 5, 5, 1, 1]$ 1×8

$[y_i] = [1, 4, 9, 4, 2, 6, 7, 3]$ 1×8

$$N_1(\xi,\eta) = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta)$$

$$N_2(\xi,\eta) = -\frac{1}{4}(1+\xi)(1-\eta)(1-\xi+\eta)$$

$$N_3(\xi,\eta) = -\frac{1}{4}(1+\xi)(1+\eta)(1-\xi-\eta)$$

$$N_4(\xi,\eta) = -\frac{1}{4}(1-\xi)(1+\eta)(1+\xi-\eta)$$

$$N_5(\xi,\eta) = \frac{1}{2}(1-\xi^2)(1-\eta)$$

$$N_6(\xi,\eta) = \frac{1}{2}(1+\xi)(1-\eta^2)$$

$$N_7(\xi,\eta) = \frac{1}{2}(1-\xi^2)(1+\eta)$$

$$N_8(\xi,\eta) = \frac{1}{2}(1-\xi)(1-\eta^2)$$

$$N_1(\xi,\eta) = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta)$$

$$\frac{\partial N_1}{\partial \xi} = -\frac{1}{4}(1-\eta)\,\frac{\partial\left[(1-\xi)(1+\xi+\eta)\right]}{\partial \xi} =$$

$$= -\frac{1}{4}(1-\eta)\left[\frac{\partial(1-\xi)}{\partial \xi}(1+\xi+\eta) + \frac{\partial(1+\xi+\eta)}{\partial \xi}(1-\xi)\right] =$$

$$= -\frac{1}{4}(1-\eta)\left[-1\cdot(1+\xi+\eta) + 1\cdot(1-\xi)\right] =$$

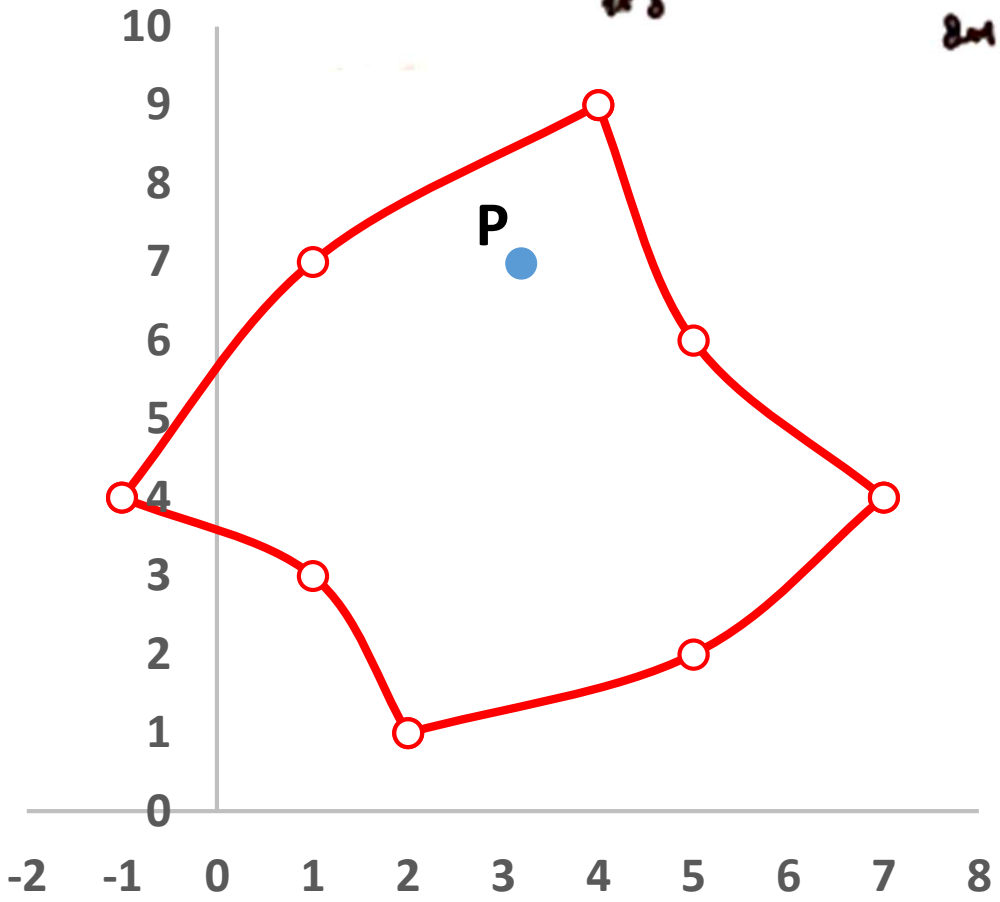
$$= -\frac{1}{4}(1-\eta)\left[-1-\xi-\eta+1-\xi\right] = -\frac{1}{4}(1-\eta)\left[-2\xi-\eta\right] =$$

$$= \frac{1}{4}(1-\eta)(2\xi+\eta)$$

i	$\frac{\partial N_i}{\partial \xi}$	$\frac{\partial N_i}{\partial \eta}$
1	$\frac{1}{4}(1-\eta)(2\xi+\eta)$	$\frac{1}{4}(1-\xi)(\xi+2\eta)$
2	$\frac{1}{4}(1-\eta)(2\xi-\eta)$	$\frac{1}{4}(1+\xi)(2\eta-\xi)$
3	$\frac{1}{4}(1+\eta)(2\xi+\eta)$	$\frac{1}{4}(1+\xi)(\xi+2\eta)$
4	$\frac{1}{4}(1+\eta)(2\xi-\eta)$	$\frac{1}{4}(1-\xi)(2\eta-\xi)$
5	$-(1-\eta)\xi$	$-\frac{1}{2}(1-\xi^2)$
6	$\frac{1}{2}(1-\eta^2)$	$-(1+\xi)\cdot\eta$
7	$-(1+\eta)\xi$	$\frac{1}{2}(1-\xi^2)$
8	$-\frac{1}{2}(1-\eta^2)$	$-(1-\xi)\cdot\eta$

$$x_p = [N(\xi_p, \eta_p)]_{i=1\ldots8} \cdot \{x_i\}_{i=1\ldots8}^{e} = 3.1925 \text{ mm}$$

$$y_p = [N(\xi_p, \eta_p)]_{i=1\ldots8} \cdot \{y_i\}_{i=1\ldots8}^{e} = 6.9761 \text{ mm}$$



$$\det[J]\big|_p = \left.\frac{\partial [N(\xi,\eta)]}{\partial \xi}\right|_p \cdot \underset{8\times 1}{\{x_i\}_e} \cdot \left.\frac{\partial [N(\xi,\eta)]}{\partial \eta}\right|_p \cdot \underset{8\times 1}{\{y_i\}_e} +$$

$$- \left.\frac{\partial [N(\xi,\eta)]}{\partial \xi}\right|_p \cdot \underset{8\times 1}{\{y_i\}_e} \cdot \left.\frac{\partial [N(\xi,\eta)]}{\partial \eta}\right|_p \cdot \underset{8\times 1}{\{x_i\}_e}$$

ξ	0.57735							
η	0.57735							
i	1	2	3	4	5	6	7	8
N	-0.09623	-0.16667	0.096225	-0.16667	0.140883	0.525783	0.525783	0.140883
dN/dξ	0.183013	0.061004	0.683013	0.227671	-0.24402	0.333333	-0.91068	-0.33333
dN/dη	0.183013	0.227671	0.683013	0.061004	-0.33333	-0.91068	0.333333	-0.24402
det[J]	**9.821367**							